Int J Theor Phys (2010) 49: 1693–1698
DOI 10.1007/s10773-010-0348-8

Higher Dimensional Cosmology with Normal Scalar Field and Tachyonic Field

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Received: 21 July 2009 / Accepted: 8 December 2009 / Published online: 15 April 2010

Abstract We have considered N-dimensional Einstein field equations in which four-dimensional space-time is described by a FRW metric and that of extra dimensions by an Euclidean metric. We have supposed that the higher dimensional anisotropic universe is filled with only normal scalar field or tachyonic field. Here we have found the nature of potential of normal scalar field or tachyonic field. From graphical representations, we have seen that the potential is always decreases with field ϕ increases.

Keywords Higher dimensions · Acceleration · Scalar field · Tachyonic field

Recently the most interesting problems of particle physics cosmology is the origin of accelerated expansion of the present universe. From astronomical observations it is evident that the present day universe has the critical energy density containing about 70% dark energy and 30% dark matter, responsible for cosmic acceleration [1]. From string/M-theory or braneworld model recently many cosmological models have been constructed by introducing quintessence, dilaton or antisymmetric tensor field. On the other hand dark energy models are basically based on holographic principle developed by AdS/CFT.

To understand the current quintessential behavior the existence of extra dimensions is very much necessary like dark energy and dark matter. Model of higher dimension was proposed by Kaluza and Klein [2, 3] who tried to unify gravity with electromagnetic interaction by introducing an extra dimension which is basically an extension of Einstein general

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relativity in 5D. The activities of extra dimensions also verified from the STM theory [4] proposed recently by Wesson et al. [5]. As our space-time is explicitly four dimensional in nature so the 'hidden' dimensions must be related to the dark matter and dark energy which are also 'invisible' in nature. To explain the nature of dark energy there are several models such as tachyon [6], phantom [7], K-essence [8], etc.

The importance of extra dimension in cosmology has been discussed by many authors. In some analysis the metric is assumed to be uniform in extra dimension [9–11]. For the metric having non-trivial dependence the extra dimension, the analysis on inflation may be altered [12, 13]. Nihei [14] has find inflationary solutions with non-trivial configurations in 5 dimensional gravity with an orbifold extra dimension S^1/Z_2. Panigrahi and Chatterjee [15] have found that the inflationary scenario is possible for inhomogeneous extra dimensional model and for the homogeneous case an initially decelerating universe starts accelerating under going a flip. Ibánez and Verdaguer [16] have obtained a set of solutions of Einstein's equation in an N-dimensional vacuum model and also homogeneous solutions with expanding 3-dimensional isotropic space. The 4D perfect fluid solutions in flat universe model have been obtained by Krori et al. [17], Gleiser and Diaz [18], using a higher dimensional anisotropic cosmology (Bianchi-I) which are compatible with contraction of all the extra dimensions. In [19] Paul has considered the theories of imperfect fluid such as Eckart, EIT, TIS and FIS to obtain the cosmological solutions for flat FRW with extra dimensions by Kasner type Euclidean metric and present an analysis of a n-dimensional vacuum Einstein's field equations. In [20] Gorbunov and Sibiryanov have proposed a cosmological model of self accelerated brane universe with warped extra dimension. In [21] Peng et al. extend the direct quantum approach to the FRW cosmology from 4D to 5D and obtained a Hamiltonian formulation for a wave like 5D FRW cosmology. In [22] Panigrahi et al. have shown a scenario in homogeneous 5D space time which behave a decelerating expansion in the early epoch along with an accelerating situation at the present line without introducing any external quintessence-like scalar field in the presence of extra dimension.

In this work, we have considered N-dimensional Einstein field equations in which 4-dimensional space-time is described by a FRW metric and that of the extra d-dimensions by an Euclidean metric. We also consider the anisotropic model of the universe is filled with normal scalar field or tachyonic field. Here in extra dimensional phenomenon we have shown the change of the potential $V(\phi)$ corresponding to the field ϕ for the normal scalar field and tachyonic field in accelerating scenario.

We consider homogeneous and anisotropic N-dimensional space-time model described by the line element

$$ds^2 = ds^2_{\text{FRW}} + \sum_{i=1}^{d} b^2(t)\, dx_i^2 \tag{1}$$

where d is the number of extra dimensions ($d = N - 4$) and ds^2_{FRW} represents the line element of the FRW metric in four dimensions is given by

$$ds^2_{\text{FRW}} = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{2}$$

where $a(t)$ and $b(t)$ are the functions of t alone represents the scale factors of 4-dimensional space time and extra dimensions respectively. Here k $(=0,\ \pm 1)$ is the curvature index of the corresponding 3-space, so that the above universe is described as flat, closed and open respectively.

The Einstein's field equations for the above non-vacuum higher dimensional space-time symmetry are

$$3\left(\frac{\dot{a}^2+k}{a^2}\right)=\frac{\ddot{D}}{D}-\frac{d^2}{8}\frac{\dot{b}^2}{b^2}+\frac{d}{8}\frac{\dot{b}^2}{b^2}+\rho \tag{3}$$

$$2\frac{\ddot{a}}{a}+\frac{\dot{a}^2+k}{a^2}=\frac{\dot{a}}{a}\frac{\dot{D}}{D}+\frac{d^2}{8}\frac{\dot{b}^2}{b^2}-\frac{d}{8}\frac{\dot{b}^2}{b^2}-p \tag{4}$$

and

$$\frac{\ddot{b}}{b}+3\frac{\dot{a}}{a}\frac{\dot{b}}{b}+\frac{\dot{D}}{D}\frac{\dot{b}}{b}-\frac{\dot{b}^2}{b^2}=-\frac{p}{2} \tag{5}$$

where ρ and p are energy density and isotropic pressure respectively. Here we choose here $8\pi G=c=1$ and $D^2=b^d(t)$, so we have $\frac{\dot{D}}{D}=\frac{d}{2}\frac{\dot{b}}{b}$ and $\frac{\ddot{D}}{D}=\frac{d}{2}\frac{\ddot{b}}{b}+\frac{d^2-2d}{4}\frac{\dot{b}^2}{b^2}$.

Hence choosing the scale factors a and b as the power of cosmic time t are given by $a=a_0t^m$, $b=b_0t^n$ where a_0, b_0, m and n are positive constants. So the field equations (3), (4) and (5) become

$$\frac{3m^2}{t^2}+\frac{3kt^{-2m}}{a_0^2}=\frac{dn(dn+n-4)}{8t^2}+\rho \tag{6}$$

$$\frac{m(3m-2)}{t^2}+\frac{kt^{-2m}}{a_0^2}=\frac{dn(dn+4m-n)}{8t^2}-p \tag{7}$$

and

$$\frac{n(dn-6m-2)}{t^2}+p=0 \tag{8}$$

Now from (7) and (8) we have the quadratic equation for n as

$$2(m^2-n^2-m+n)-\frac{d+12}{2}mn-\frac{d^2+7d-16}{8}n^2+m^2+\frac{k}{a_0^2}t^{-2m+2}=0 \tag{9}$$

From the above equation, we conclude that either $k=0$ or $m=1$. So for non-flat universe, we must have $m=1$. To solve this equation we fix the value of m as 1 and we get the expression for n as

$$n=\frac{-2(d+8)a_0+2\sqrt{2d(d+7)k+\{64+3d(d+10)\}a_0^2}}{d(d+7)a_0} \tag{10}$$

We now consider the universe is filled with normal scalar field or tachyonic field. For these two cases we analyze the behavior of the universe in extra dimension.

Normal Scalar Field

The energy density and pressure due to normal scalar field ϕ are given by

$$\rho=\frac{1}{2}\dot{\phi}^2+V(\phi) \tag{11}$$

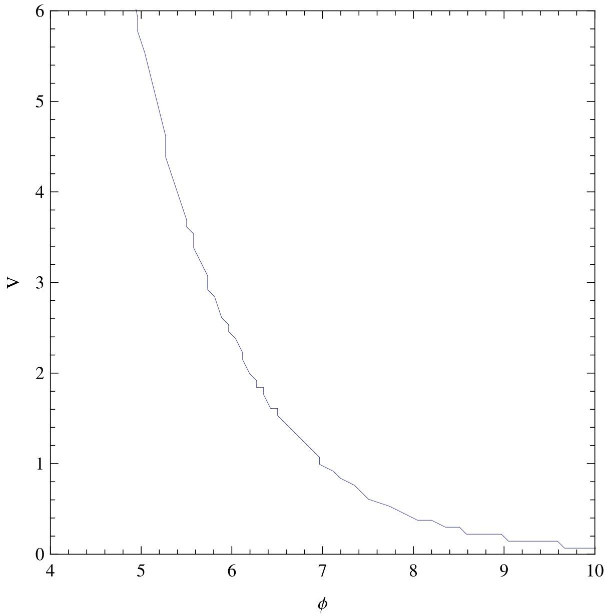

Fig. 1 The variation of V against ϕ (with $m=1, a_0=1, d=5, k=1$) for normal scalar field

and

$$p=\frac{1}{2}\dot{\phi}^2-V(\phi) \tag{12}$$

where $V(\phi)$ is the relevant potential for normal scalar field ϕ.

Using (6)–(7), we can find the expressions for $V(\phi)$ and ϕ as

$$V=\frac{(4m-dn)(dn+6m-2)}{8t^2}+\frac{2k}{a_0^2}t^{-2m} \tag{13}$$

and

$$\phi=\frac{1}{2}\int t^{-m-1}\sqrt{\frac{8kt^2+\{-d(n-2)n+2m(dn+4)\}a_0^2t^{2m}}{a_0^2}}\,dt \tag{14}$$

From above forms of V and ϕ, we see that V can not be expressed explicitly in terms of ϕ.

For $k=0$, $V(\phi)$ can be expressed in terms of ϕ as

$$V(\phi)=\frac{1}{8}\{24m^2+dn(2-dn)-2m(4+dn)\}\,e^{-\frac{4\phi}{\sqrt{dn(2+(d-2)n)+2m(4+dn)}}} \tag{15}$$

For $k\neq 0$ (i.e., $m=1$), $V(\phi)$ can be expressed in terms of ϕ as

$$V(\phi)=\frac{1}{8}\left(16-d^2n^2+\frac{16k}{a_0^2}\right)e^{-\frac{4a_0\phi}{\sqrt{8k+(8+dn(4+(d-2)n))a_0^2}}} \tag{16}$$

From (15) and (16), we see that $V(\phi)$ is exponentially decreasing function of normal scalar field ϕ for both flat ($k=0$) and non-flat models ($k\neq 0,\ m=1$). Figure 1 shows the variation of $V(\phi)$ against normal scalar field ϕ.

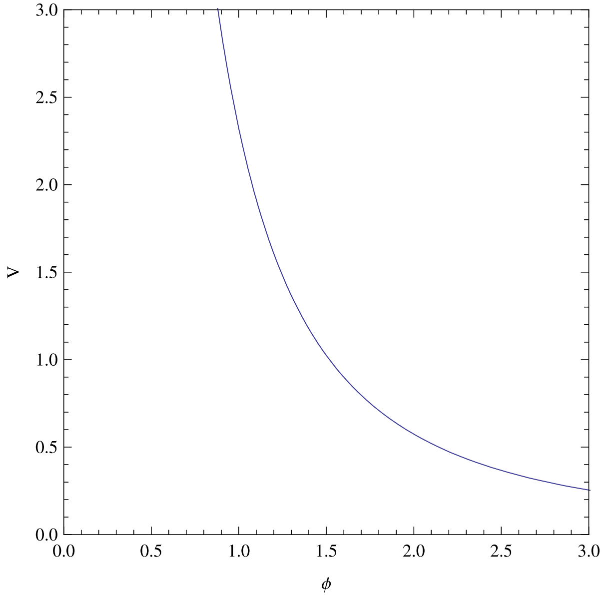

Fig. 2 The variation of V against ϕ (with $m=1, a_0=1, d=5, k=1$) for tachyonic field

Tachyonic Field

The energy density ρ and pressure p due to the tachyonic field ϕ are given by

$$\rho = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} \tag{17}$$

and

$$p = -V(\phi)\sqrt{1-\dot{\phi}^2} \tag{18}$$

where $V(\phi)$ is the relevant potential for the tachyonic field ϕ. It can be seen that $\frac{p_\phi}{\rho_\phi} = -1+\dot{\phi}^2 > -1$ for normal tachyon. Using (6)–(7), we can find the expressions for $V(\phi)$ and ϕ as

$$V(\phi) = \frac{1}{8a_0^2}t^{-2(m+1)} \times \sqrt{\{8kt^2+(-d(n-2)n+2m(dn+4))a_0^2t^{2m}\}\{24kt^2+(24m^2-dn(dn+n-4))a_0^2t^{2m}\}} \tag{19}$$

and

$$\phi = \int \frac{\sqrt{2}\sqrt{8kt^2+(-d(n-2)n+2m(dn+4))a_0^2t^{2m}}}{\sqrt{24kt^2+(24m^2-dn(dn+n-4))a_0^2t^{2m}}}\,dt \tag{20}$$

From above forms of V and ϕ, we see that V can not be expressed explicitly in terms of ϕ.

For $k=0$, $V(\phi)$ can be expressed in terms of ϕ as

$$V(\phi)=\frac{\{dn(2-n)+2m(4+dn)\}\sqrt{24m^2-(d-1)dn^2-4m(4+dn)}}{4\sqrt{24m^2-dn(dn+n-4)}\,\phi^2} \tag{21}$$

For $k\neq 0$ (i.e., $m=1$), $V(\phi)$ can be expressed in terms of ϕ as

$$V(\phi)=\frac{\{8k+(8-d(n-4)n)a_0^2\}\sqrt{8k+(8+dn(n-dn-4))a_0^2}}{4a_0^2\sqrt{24k+(24-dn(n+dn-4))a_0^2}\,\phi^2} \tag{22}$$

From (21) and (22), we see that $V(\phi)$ is inverse square (decreasing) function of tachyonic field ϕ for both flat ($k=0$) and non-flat models ($k\neq 0,\ m=1$). Figure 2 shows the variation of $V(\phi)$ against tachyonic field ϕ.

In this work, we have considered $N\ (=4+d)$-dimensional Einstein field equations in which 4-dimensional space-time is described by a FRW metric and that of the extra d-dimensions by an Euclidean metric. We also consider the anisotropic model of the universe is filled with normal scalar field or tachyonic field. We have shown that the model is valid either for $k=0$ (flat) or for $k\neq 0$ (with $m=1$) (non-flat). For both flat and non-flat cases, we have shown that the potential is exponentially decreasing function of normal scalar field and also the potential is inverse square (decreasing) function of tachyonic field. Here in extra dimensional phenomenon we have shown the change of the potential $V(\phi)$ corresponding to the field ϕ for the normal scalar field and tachyonic field in accelerating scenario. From Figs. 1 and 2, it has been seen that the potential functions corresponding to normal scalar field and tachyonic field are always decreases with their fields for $m=1$, $a_0=1$, $d=5$, $k=1$.

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